

The Origin of Cosmic Rays – New Interest in an Old Question

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To KONRAD BLEULER on the occasion of his 60th birthday

After briefly reviewing the experimental facts that seem of direct relevance for the question of the origin of cosmic rays, and after quickly going through the “classical” theories of cosmic ray origin, we discuss in some detail those theories that have been put forward over the past few years since the discovery of pulsars. This discovery brought the first observational evidence for the existence of super-strong magnetic fields in nature.

Introduction

Ever since the discovery of cosmic rays at the beginning of this century has the question of their origin been especially intriguing. Today this is even more so because of the progress made in cosmic ray physics during the past one or two decades. There are three main reasons for this new interest:

(i) Space research has opened up new experimental possibilities by using rockets, satellites, and deep-space probes to measure the cosmic ray flux outside the earth’s atmosphere. A wealth of new experimental data is coming from these measurements.

(ii) Radio astronomy has proved to be a powerful means to detect relativistic particles through their non-thermal radiation. This is possible even for the most distant objects so far detected in the universe, the quasi-stellar objects. This evidence, strictly speaking, only proves the existence of relativistic electrons (or positrons), which are detected through their magneto-bremsstrahlung or inverse Compton emission. The first process is the important one in the radio range, whereas in the optical and X-ray range the second one may be of equal or larger importance.

(iii) As soon as quantitative estimates of energy densities became available for cosmic rays in the Galaxy as well as in external systems, it was realized that cosmic rays are a major constituent of the universe as far as energy is concerned, despite their

very small number densities. It indeed turned out that the energy density of cosmic rays in the Galaxy, the energy density of the total light of the stars, the energy density of the interstellar magnetic field, as well as the energy density of the thermal gas in the Galaxy, all have the same order of magnitude, namely $\sim 1 \text{ eV/cm}^3$. This immediately illustrates that, in addition to the nuclear processes taking place in the interior of stars, that balance the radiative losses at the stars’ surfaces, there must be a second, equally important energy transfer mechanism that leads to the generation of large amounts of relativistic particles.

In the following three subsections we shall in turn briefly discuss the observational data as far as they appear of direct relevance to the origin of cosmic rays, some of the classical” theories of the origin of cosmic rays, and then turn to new theories that were put forth over the past three years since the discovery of pulsars.

Observations

The main properties characterizing the cosmic rays are

- (i) their total intensity,
- (ii) the energy spectra of their various constituents,
- (iii) their chemical composition, including isotopic abundance ratios.

A fourth property may be added to this, i. e., their isotropy, or rather the upper limits that presently exist for the maximum degree of anisotropy for cosmic rays of different energies.

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At present all the detailed observational evidence rests on earth-bound or near-earth experiments. In the future it might be possible, however, to study, e.g., the chemical composition of cosmic rays in distant sources through gamma-ray spectroscopy. The disadvantages of earth-bound or near-earth measurements might then be overcome, which are due to the fact that neither the total intensity nor the individual energy spectra nor the chemical composition of cosmic rays remain unchanged while the particles propagate from the source to the observer. One may distinguish three physically different regions where these modifications occur, (i) in interstellar space through the interaction with the tenuous weakly ionized interstellar gas and the interstellar magnetic field, (ii) in interplanetary space through the interaction with the solar wind, and finally, (iii) as far as earth-bound observations are concerned, the earth atmosphere is strongly interacting with the incoming cosmic rays, thus changing their primary properties.

For the question of the origin of the cosmic rays it is most important that one is able to reconstruct the properties at the source from the properties measured at the earth. Over the past few years considerable progress has been made in this field.

We shall ignore here the methods applied to correct for atmospheric modulations. As far as the interaction with the solar wind is concerned which considerably modifies the energy spectra of cosmic rays below a few hundred MeV, new calculations taking into account adiabatic expansion effects^{1, 2} have considerably improved the earlier models by PARKER³. These calculations clearly demonstrate that it will unfortunately not be possible to recover the interstellar flux of low energy cosmic rays ($E \lesssim 100$ MeV) which would be particularly interesting for some astrophysical applications.

As far as the penetration of cosmic rays through interstellar space is concerned, there are the interactions with the interstellar gas particles as well as the interaction with the interstellar magnetic fields that matter. While the first class of processes is most important for cosmic ray nuclei, the second process is the dominant loss mechanism for electrons (and positrons). In the interaction with the interstellar gas, cosmic ray nuclei lose energy due to ionisation losses as well as due to nuclear interactions. In particular this second process can now be studied in much greater detail and much more ac-

curately than has been possible a few years ago, since a number of new cross-sections have become available. Results from solving a Fokker-Planck-equation for the propagation of cosmic rays through interstellar space, taking into account various interaction processes, have been given in an extensive review article by SHAPIRO and SILBERBERG⁴. These results, which have largely been confirmed by calculations of other groups, show that the chemical composition of cosmic rays at the sources must be very similar to the chemical composition found in the sun and in other stars as well as in meteorites. This puts severe constraints on all models trying to explain the origin of cosmic rays. In the last section of this paper we shall discuss models for a pulsar origin of cosmic rays. Pulsars are taken to be rotating magnetic neutron stars. However, as pointed out, e.g. by ROSEN⁵, one would expect that very shortly after the formation of a neutron star ($\sim 10^3$ sec) its atmosphere would completely be dominated by iron.

It must be stressed though that despite the fact that the total flux of cosmic rays is now known up to energies of $\sim 10^{19}$ eV (c.f. Fig. 1, adapted from MEYER⁶), and possibly even $4 \cdot 10^{21}$ eV-particles have been detected⁷, all the information concerning e.g. the chemical composition of cosmic rays is so far restricted to particles with energies $E \leq 10^{12}$ eV. These particles are the dominant ones, of course, with only a fraction of 10^{-4} being at higher ener-

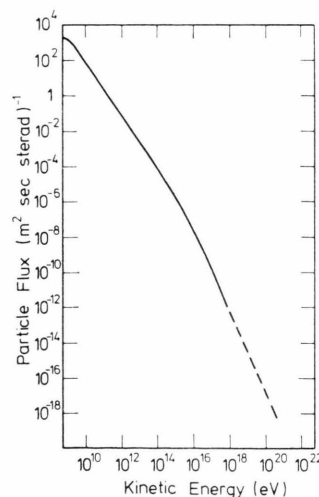


Fig. 1. The energy spectrum of the total cosmic radiation. The solid part of the curve was adapted from MEYER⁶, the dashed part is an extrapolation based on more recent measurements.

gies. For theories of the origin of cosmic rays on the other hand, one is particularly interested in the high energy tail and it would be most valuable if their chemical composition as well as their degree of isotropy could be determined. At present this is extremely difficult if not impossible because the small particle flux requires large detectors and/or prohibitively long integration times.

“Classical” Theories on the Origin of Cosmic Rays

As ‘classical’ theories on the origin of cosmic rays we shall denote all those that do not make use of superstrong electromagnetic fields, the existence of which was unknown prior to the discovery of pulsars⁸. The most extensive discussion of various classical theories of the origin of cosmic rays may still be found in the book by GINZBURG and SYROVATSKII⁹, though a number of more recent reviews (e. g.¹⁰) do exist.

There are two fundamentally different schools of thought, one of which assumes the cosmic rays to be of galactic origin, whereas the other assumes an extragalactic origin of cosmic rays. At present there is no conclusive evidence against either of these hypotheses. In the future one might hope, however, to decide this question experimentally by measuring the abundance ratios of some of the unstable isotopes with medium and long decay rates.

If the cosmic rays are indeed extragalactic in origin there is a serious energy problem. There are a number of reasons why the present energy density of $\sim 1 \text{ eV/cm}^3$ should represent a stationary state. E. g. from measurements in meteorites one knows that there was no major change over the past $\sim 10^9$ years.. However, to maintain this energy density universally despite the expansion of the universe, requires a cosmic ray production rate of $\sim 5 \cdot 10^{-30} \text{ erg/cm}^3 \text{ sec}$ even if we neglect all other loss mechanisms. It has been suggested that cosmic rays are generated in strong non-thermal sources such as radiogalaxies and quasars. If this were so, each of these sources would have to generate $\sim 10^{49} \text{ erg/sec}$ in the form of relativistic particles¹¹, whereas their energy output in the entire electromagnetic spectrum is $\leq 10^{47} \text{ erg/sec}$.

There are further difficulties with an extragalactic origin of cosmic rays which, however, strictly speaking refer only to the electron component. First,

the universal blackbody radiation will severely limit the lifetime of relativistic electrons due to inverse Compton losses, second, as was first noted by FELTEN and MORRISON¹² the Compton scattering would produce an X-ray background that exceeds the observed one.

Here we shall concentrate on the models suggesting a galactic origin of cosmic rays. There are again two classes of models which may be called the point-source models and statistical models. We shall discuss them in turn.

There are a few classes of astronomical objects that suggest themselves for point-source models for the origin of cosmic rays. First of all there are of course the supernovae, which are known to liberate vast amounts of energy, gravitational in origin ($\geq 10^{53} \text{ erg}$). Hydrodynamic calculations of a supernova collapse by COLGATE and his co-workers suggest that a sufficiently large fraction of the original mass of the star will indeed be accelerated to relativistic energies (e. g.¹³). The results depend critically on what fraction of the energy originally carried away by neutrinos is reabsorbed and then carried outward through the star by a shock-wave. Quantitative estimates suggest that with a supernova rate of one or two per century one could indeed balance the energy loss of the cosmic rays that is of the order of $\sim 10^{40} \text{ erg/sec}$ both for the nuclei and the electrons.

Other models suggesting a localized origin of cosmic rays consider e. g. novae or some activity in the galactic center region. They are extensively discussed e. g. in Reference⁹. We mention here only two more recent models which are due to HAVNES¹⁴ who suggests that low-energy cosmic rays are produced by magnetic A-stars which have surface magnetic fields of 10^3 Gauss in which interstellar particles could be accelerated, and another model due to COSWICK and PRICE¹⁵, who suggest that white dwarfs, some of which may also possess strong surface magnetic fields, could generate cosmic rays. This model has recently however been criticized¹⁶ as being in conflict with the observations of the magnetic fields of white dwarfs and current theories on magnetic field decay.

There remains the discussion of statistical acceleration processes. There are two basically different forms of electromagnetic acceleration mechanisms considered, (i) betatron acceleration in a homogeneous magnetic field that increases with time,

and (ii) the acceleration of particles in collisions with moving magnetic field inhomogeneities.

In the betatron process a change in the magnetic field causes a change in those velocity components of a particle that are orthogonal to the field. Using the fact that for slowly varying fields the magnetic moment μ is an invariant, we find

$$\frac{1}{2} m v_{\perp}^2 = \mu \mathbf{B}. \quad (1)$$

This relation shows that an increase of the field \mathbf{B} will cause an increase in v_{\perp}^2 and a decrease will cause a corresponding energy loss. A net energy gain is then achieved by the fact that during the time that the particle takes to travel from a region of increasing magnetic field into a region where the field is decreasing a redistribution of the momentum from v_{\perp} to v_{\parallel} occurs due to collisions.

The second statistical mechanism that has first been proposed by FERMI¹⁷ considers the energy change of particles reflected by moving magnetic walls. This change may either be calculated by using the energy and momentum conservation in the frame of the moving wall or by calculating the work done by the induced electric field. The change of energy turns out to be

$$\Delta E = (-2 E/c^2) (\mathbf{u} \cdot \mathbf{v}) \quad (2)$$

where \mathbf{v} is the particle's velocity, E its energy and \mathbf{u} the velocity of the moving field inhomogeneity. Equation (2) shows that the energy will increase in head-on collisions, and will decrease by the same amount in overtaking collisions (always $|\mathbf{v}| \gg |\mathbf{u}|$). A net energy gain is in this case due to the fact that head-on collisions occur statistically more often than overtaking collisions. Weighting the energy changes by the probability of the corresponding collisions one finds

$$\overline{\Delta E} = \zeta (u^2/c^2) E \quad (3)$$

where $\zeta = 2$ for $\mathbf{v} \parallel \mathbf{u}$ and $\zeta \approx 4/3$ for an isotropic velocity distribution. To quote numbers let us take interstellar clouds to represent the moving magnetic field inhomogeneities. The mean distance between such clouds in interstellar space is $l \approx 3 \cdot 10^{20}$ cm, their mean random velocity $|u| \approx 10^6$ cm/sec. Travelling with velocity v a particle will hit such a cloud once in $\tau = l/v$. The rate of change of energy is then given by

$$\frac{dE}{dt} = \zeta \frac{u^2}{c l} \frac{v}{c} E. \quad (4)$$

Assuming the particle has an initial energy of 10^{12} eV already, we find with the values given above $dE/dt \approx 10^{-7}$ eV/sec or ~ 3 eV/year. Actually, at low energies the gain rate would be smaller than the energy loss rate caused by ionisation losses so that the process will work only if the particles have a sufficiently high energy to start with.

Though the smallness of the net energy change per collision may be overcome by the long cosmical timescales, there are considerable limitations on the maximum obtainable energies irrespective of this. These arise from the fact that both in the case of the betatron process and in the case of collisions with moving magnetic walls the Lamor-radius of the particles $R_L \approx E/qH$, where q is the charge of the particles, must remain smaller than the dimensions over which the field varies.

Pulsar-Origin of Cosmic Rays

The discovery of pulsed radio sources in 1968⁸ has not only for the first time brought supporting evidence for the existence of neutron stars that were discussed by theoreticians since the early 1930's, but has also at the same time greatly stimulated new theories of the origin of cosmic rays.

The observational data, in particular the great regularity and the shortness of the pulse repetition period which is typically of the order of one second, require an underlying clock mechanism which could either be the pulsation or rotation of a highly condensed object with mass density $\rho \geq 10^{10}$ g cm⁻³. The only stable macroscopic configurations known to exist in this density regime are neutron stars, and a number of arguments can be put forward why the clock mechanism of pulsars is the rotation of a neutron star.

It should be stressed, however, that despite the evidence for the existence of neutron stars the details of how they form are still unclear. This is due to the fact that so far no method has been found which would allow to extend current stellar evolution calculations beyond the hydrogen and helium burning phases in an entirely selfconsistent manner. It was already suspected in the 1930's on energetic grounds that neutron stars will form during supernova explosions, but only during the past decade have hydrodynamic calculations become available [e.g.¹³] that allow the calculation of some of the details of this formation process.

Despite the gap that remains in our ability to simulate in stellar evolution calculations the evolutionary path of a star from its initial formation to its ending as a neutron star, there can be little doubt that during all intermediate steps the electrical conductivity of the stellar material will remain sufficiently high to prevent any primeval stellar magnetic field from decay, unless there are loss mechanisms other than ohmic dissipation. The important consequence of the magnetic flux conservation argument is that neutron stars should possess enormous magnetic fields since $B_f = B_i (R_i/R_f)^2 \approx 10^{10} B_i$ (i = initial, f = final), where B_i would typically be ≥ 1 Gauss.

The pulsar observations indeed seem to prove the existence of such strong surface magnetic fields. We refer to the fact that approximately 40% of all pulsars show a secular increase in their pulse period with $4 \cdot 10^{-13} \leq \dot{P} \leq 10^{-16}$, the lower value representing the present limit of detectability. Such a period increase is to be expected for a rotating star surrounded by a strong surface magnetic field, the axis of which is inclined with respect to the rotation axis of the star (angle α)

$$-dE/dt = (2/3 c^3) \mathbf{m}_{\perp}^2 \Omega^4 \quad (5)$$

where $\mathbf{m}_{\perp} = \mathbf{m} \sin \alpha$ is the orthogonal component of the magnetic moment \mathbf{m} , and Ω the angular frequency of the star. This energy loss must be balanced by a change of the rotational energy of the system

$$-\left(\frac{dE}{dt}\right) = \frac{d}{dt} \left(\frac{I}{2} \Omega^2 \right), \quad (6)$$

I being the moment of inertia of the star.

Equating (5) and (6), inserting the magnetic moment \mathbf{m} of a dipole, and introducing $P = 2\pi/\Omega$ we find

$$B_s^2 \sin^2 \alpha = (6 c^3 / 4 \pi^2) I (P \dot{P}) / R_s^6. \quad (7)$$

B_s denotes the surface magnetic field. Neutron star calculations show I to be in the range $5 \cdot 10^{43} \leq I \leq 10^{45}$ g cm². From pulsar observations we have $0.033 \leq P \leq 3.74$ sec, and $4 \cdot 10^{-13} \leq \dot{P} \leq 10^{-16}$. Inserting the actual values, we find (for $\sin \alpha = 1$):

$$10^{11} \leq B_s \leq 10^{13} \text{ Gauss.}$$

With the model assumptions stated we can at least verify the magnetic field strengths expected on the basis of the flux conservation argument. In what

follows we shall assume that such strong surface magnetic fields do exist in pulsars and, furthermore, that they may be treated in a first approximation as simple dipole fields.

It is immediately evident that these strong fields open new possibilities for the generation of cosmic rays. First, because of the rapid rotation of the star, 30 times per second in the case of the so-called Crab pulsar, the magnetic fields will induce almost equally strong electric fields

$$\mathbf{E} = - (1/c) (\mathbf{r} \times \boldsymbol{\Omega}) \times \mathbf{B}. \quad (8)$$

With $10^{10} \leq |E| \leq 10^{12}$ V/cm in a linear acceleration process, particles could reach cosmic ray energies since $|E| \ll 1$ V/cm in intersellar space, provided, however, that favourable particle trajectories exist along which charged particles can traverse the corresponding potential difference. This point is not clear at present since, as was first pointed out by GOLDBREICH and JULIAN¹⁸, the induced electric field will cause charged particles to be pulled out of the surface of the star, leading to the formation of a dense magnetosphere. Though some attempts^{18, 19} have been made to describe what changes would be imposed upon the simple dipole solution for the electromagnetic fields by the presence of a magnetosphere, no final self-consistent solution has yet been found.

Leaving aside for the moment the still-unsolved magnetospheric problems and the possibility of particle acceleration from the surface of a neutron star, there is a second particle acceleration mechanism which follows from the "oblique rotator" model for pulsars. The rotating magnetic dipole will, if its axis is inclined with respect to the rotation axis of the star, generate intense low-frequency electromagnetic waves. As can be seen by evaluating Eq. (5), these carry away an enormous amount of energy, in the case of the already-mentioned "Crab pulsar":

$$dE/dt > 10^{38} \text{ erg/sec.}$$

As was first noted by GUNN and OSTRICKER^{20, 21} charged particles will couple very efficiently to these low frequency waves, obtaining energies well in excess of their rest mass energy. All theoretical calculations of this process so far ignore the effects close to the surface of the star, i. e. possible magnetospheric effects, but assume the electromagnetic fields in the wave-zone to be those of a rotating mag-

netic dipole in vacuum²². The first analytical and numerical studies^{20, 21, 23-25} considering only the pure wavefield terms of the electromagnetic fields already showed that with conditions as given in the Crab pulsar, protons and electrons can reach energies $\sim 10^{13}$ and $\sim 10^{14}$ eV respectively. These energies just satisfy the energy requirements for those particles that give rise to the continuous X-ray emission of the Crab nebula.

Since those early calculations cited above, considerable progress has been made in studying the particle motion in intense low-frequency electromagnetic waves. This acceleration process turned out to be even more efficient than has previously been anticipated. In the following we shall report these new results.

The interaction between particles and fields is characterized by the following dimensionless Lorentz-invariants

$$\begin{aligned} x_0 &= (e/mc\Omega) \cdot (\mathbf{E}^2)^{1/2}, \\ h &= (e/mc\Omega) \cdot (\mathbf{E} \cdot \mathbf{B})^{1/2}, \\ g &= (e/mc\Omega) \cdot (\mathbf{E}^2 - \mathbf{B}^2)^{1/2}, \\ \chi &= (\Omega l_0/c) \cdot R \end{aligned} \quad (9)$$

where \mathbf{E} is the electric, \mathbf{B} the magnetic field strength, Ω the rotation rate of the star, R the radiation-reaction rate with $R = (e/mc\Omega)^2 \cdot [(\gamma \mathbf{E} + \mathbf{u} \times \mathbf{B})^2 - (\mathbf{E} \times \mathbf{u})^2]$, $l_0 = 2e^2/3mc^2$, and \mathbf{u} is the space-part of the four-velocity. For an oblique rotator²² with the axis being exactly orthogonal to the rotation axis of the star we find

$$\begin{aligned} x_0^2 &= (f_0^2/q^2) (\cos^2 \theta \cos^2 \Phi + \sin^2 \Phi), \\ g^2 &= (2f_0^2/q^4) (\cos^2 \theta \cos^2 \Phi - \sin^2 \Phi (2 \sin^2 \theta - 1)) \\ h^2 &= (f_0^2/q^4) (q_s + 1/q) \cos \theta \end{aligned} \quad (10)$$

where

$$f_0 = q_s^3 (e B_s / mc\Omega) \text{ and } q = \Omega r / c. \quad (11)$$

The suffix *s* denotes the parameters at the surface of the star.

The earlier calculations cited above [e. g. ²⁵] referred to $g^2 = h^2 = 0$ (plane wave). Also the case $h^2 = 0$ has been considered²⁶, however only in a qualitative manner. Here we discuss the general case.

The equation of motion for a charged particle moving in an intense electromagnetic field may be written as

$$D\mathbf{u}/d\xi = (\mathbf{e} \gamma + [\mathbf{u} \times \mathbf{b}]) + \mathbf{u} \chi \quad (12)$$

$$\text{and} \quad d\gamma/d\xi = (\mathbf{e} \cdot \mathbf{u}) + \gamma \chi \quad (13)$$

where $D\mathbf{u}/d\xi$ denotes the covariant derivative of the space components of the particle's four-velocity with respect to the dimensionless proper time $\xi = \Omega \tau$. γ denotes the energy of the particle in units of its rest mass energy, and \mathbf{e} and \mathbf{b} are proportional to the electric and magnetic fields respectively,

$$\mathbf{e} = (e/mc\Omega) \mathbf{E} \quad \text{and} \quad \mathbf{b} = (e/mc\Omega) \mathbf{B},$$

where for \mathbf{E} and \mathbf{B} we shall use the field as given by DEUTSCH²². The last terms on the right hand side in Eqs. (12) and (13) denote the radiation-reaction terms.

We first consider Eqs. (12) and (13) without the radiation-reaction terms. In this case the differential equations can be solved analytically by noting [see e. g. Ref. ²⁵] that it completely suffices to consider the initial part of the motion during which the electromagnetic fields do not change appreciably, i. e., $q \approx q_0$, $\theta \approx \theta_0$, $\Phi \approx \Phi_0$. This is due to the fact that the coupling between the particles and the wave is essentially instantaneous. For particles starting at rest, i. e. $\mathbf{u}(0) = 0$ we then find

$$\begin{aligned} (\alpha^2 + \beta^2) \gamma &= (x_0^2 + \beta^2) \cosh(\alpha \xi) \\ &\quad + (\alpha^2 - x_0^2) \cos(\beta \xi), \end{aligned} \quad (14)$$

$$\begin{aligned} (\alpha^2 + \beta^2) \mathbf{u} &= [\cosh(\alpha \xi) - \cos(\beta \xi)] \mathbf{e} \times \mathbf{b} \\ &\quad + [\alpha \sinh(\alpha \xi) + \beta \sin(\beta \xi)] \mathbf{e} \\ &\quad + [\beta \sinh(\alpha \xi) - \alpha \sin(\beta \xi)] \mathbf{b} \end{aligned} \quad (15)$$

$$\begin{aligned} \text{where} \quad \alpha^2 &= 0.5 (g^4 + 4h^4)^{1/2} + 0.5 g^2, \\ \beta^2 &= 0.5 (g^4 + 4h^4)^{1/2} - 0.5 g^2. \end{aligned} \quad (16)$$

It should be noted that this solution does not hold for the singular case of a particle moving exactly in the equatorial plane ($\theta = \pi/2$), or more generally the case $h^2 = 0$, $g^2 < 0$. In this case which we have also treated, the acceleration takes place over a distance $q \approx f_0^{1/2}$ which is very large.

The analytic solution shows that irrespective of the initial conditions, the particles are accelerated extremely efficiently. They obtain an energy $m_0 c^2 \gamma$ with $\gamma \approx \alpha \Delta q \approx \alpha q_0$. This result has fully been reproduced by numerically integrating Eqs. (12) and (13). Indeed we find that the final energy of a particle can be fitted to

$$\gamma_{\text{final}} = \alpha q_0 \text{ const} \quad (17)$$

the constant being equal to unity within several decimal places. However, in the singular case $\theta = \pi/2$, $g^2 < 0$ which was mentioned before, we find $\gamma = 2.5 f_0^{1/2}$. For comparison we should note that in the

case of a pure wave ($g^2 = h^2 = 0$) the results were $\gamma = (4.5 f_0^2)^{1/2}$ for particles starting on top of the wave²¹, and $\gamma = (40 f_0^2 \varrho_0^3)^{1/2}$ for particles starting at zero amplitude²³. The above results thus show that the inclusion of the near-field components makes the CCFA-mechanisms²⁵ even more efficient than has been considered in the past. This statement remains true even if we include the radiation-reaction terms in the equation of motion. This will be considered next.

So far we did not succeed in solving the full set of differential Eqs. (12) and (13) with the inclusion of the radiation-reaction terms analytically. We did succeed however in solving this set of simultaneous differential equations numerically for a variety of initial conditions. Some of the results are shown in Figs. 2–5, which all refer to particle motion in the radiation field of a rotating magnetic dipole that is at ninety degrees with respect to the rotation axis of the star.

In Fig. 2 we have chosen a “standard set” of initial conditions with $f_0 = (e B_s / m c \Omega) \varrho_s^3 = 10^{15}$, and a particle starting at rest $u_\varrho(0) = u_\theta(0) = u_\varphi(0) = 0$, $\gamma(0) = 1$ at $\varrho = \varrho_0 = 100$. The polar angle at injection was chosen to be $\theta(0) = \pi/4$. Curves labelled (a), (b), (c) show how γ , u_θ , and u_φ vary as the particle is accelerated away from the star in essentially radial direction. It should be noted that the initial part of the motion where $\varrho \approx \varrho_0$ is not shown here. For comparison we also have included curves

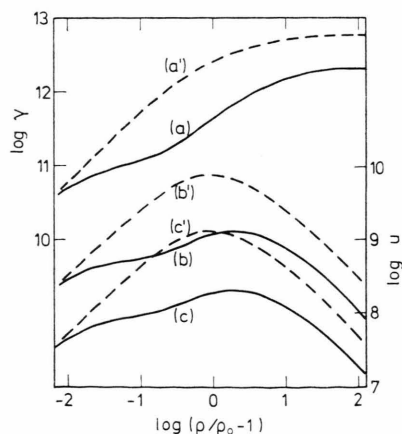


Fig. 2. The variation of γ , u_θ , and u_φ (curves a, b, c respectively) as a function of $\log(\varrho/\varrho_0 - 1)$ for a particle moving in a radiation field characterized by $f_0 = 10^{15}$. Other initial conditions are given in the text. Curves (a'), (b'), (c') show the variation of the same quantities neglecting radiation-reaction.

(a'), (b'), (c') which show the variation of the same quantities in the absence of radiation-reaction. Without discussing the differences in detail, it is interesting to note that the influence of radiation-reaction is minor. Considering in particular the final energy the particle will reach, we see that radiation-reaction causes a reduction in γ from $6.1 \cdot 10^{12}$ to $2.2 \cdot 10^{12}$ for the initial conditions chosen above. The particle would thus still reach an energy in excess of 10^{21} eV.

In Fig. 3 we have plotted the variation of the maximum obtainable energy γ_{\max} for particles injected at different radii ϱ_0 . The numerical calculations were actually carried to smaller values of ϱ_0 than shown in the graph, but no significant change was found in γ_{\max} .

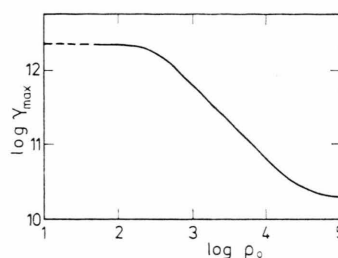


Fig. 3. The final energy a particle reaches, γ_{\max} , is plotted as function of $\log \varrho_0$, measuring the injection distance from the star. All other initial conditions are the same as for Fig. 2.

In Fig. 4 the same set of initial conditions was chosen as for Fig. 2 except that the polar angle at injection $\theta(0)$ was varied through ninety degrees.

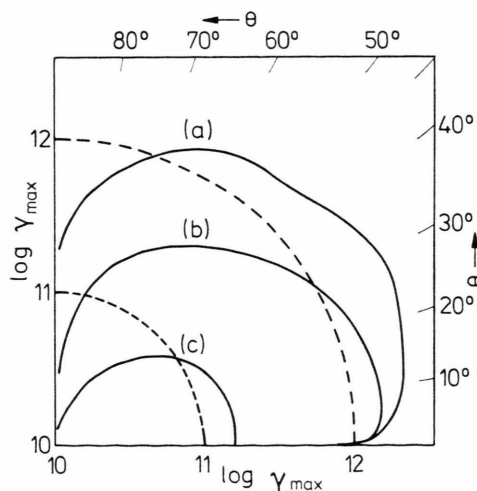


Fig. 4. Dependence of the maximum obtainable energy for the initial conditions chosen (see text) as function of the polar angle at injection $\theta(0)$. Curves (a), (b), (c) refer to $\varrho_0 = 10^2, 10^3, 10^4$ respectively.

The three curves refer to $\varrho_0 = 10^2$, 10^3 , and 10^4 respectively, again showing that particle injection closer to the star is more favourable. The variation of γ_{\max} with $\theta(0)$ is particularly interesting for the radiational aspects of this acceleration mechanism which we will, however, not discuss here.

In Fig. 5 we have plotted the variation of γ_{\max} as function of f_0 both for particle motion with (curve

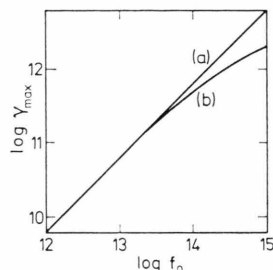


Fig. 5. The final energy that particles reach in a radiation field characterized by f_0 is plotted as function of f_0 . Curve a refers to the case without radiation-reaction, curve b includes the radiation-reaction effects. The initial conditions chosen were $\varrho_0 = 100$, $\theta(0) = \pi/4$, $\varphi(0) = 0$, $\mathbf{u}(0) = 0$.

b) and without (curve a) radiation-reaction. The two curves clearly show that radiation-reaction effects become important only for very large values of f_0 . The energy reduction due to radiation-reaction is however never so severe as to exclude energies well in excess of 10^{21} eV.

In summarizing, two major conclusions can be drawn from the above results:

1. The radiation-reaction terms in Eqs. (12) and (13) change the details of the particle motion considerably, the final energy a particle can reach is affected however only mildly. The reduction factors vary between 1 and 10 depending on the actual set of initial conditions.

2. Under favorable initial conditions even $f_0 = 10^{15}$, which for a neutron star spinning at a rate of 10^4 would correspond to $B_s \approx 10^{16}$ Gauss, leads to a maximum energy of $> 10^{21}$ eV for protons. This result must be compared with the previous results²⁰ where $f_0 \geq 10^{19}$ was required to obtain such energies.

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